

On Thermohaline Mixing in Accreting White Dwarfs

Detlev Koester

Institut für Theoretische Physik und Astrophysik, Universität Kiel, Germany;
koester@astrophysik.uni-kiel.de

Abstract. We discuss the recent claim that the thermohaline (“fingering”) instability is important in accreting white dwarfs, increasing the derived accretion fluxes potentially by orders of magnitude. We present an alternative view and conclude that at least in the steady state this is not the case and the current method of estimating accretion fluxes is correct.

1. Introduction

The thermohaline (saltfinger, fingering, double-diffusive) instability is well-known in Oceanography: a warm layer of saltwater on top of a cold body of freshwater may be dynamically stable but may nevertheless lead to complete mixing, if the diffusion of heat is faster than that of salt. In the case of stars, the rôle of salt is played by the molecular weight. A layer with higher molecular weight on top of a layer with smaller weight may be dynamically stable (no convection), but subject to a similar double-diffusive instability. Classical papers in the astrophysical context are Ulrich (1972) and Kippenhahn et al. (1980, =KRT80).

2. The instability in the scenario of KRT80

The instability starts from a boundary layer separating the two layers. A front of a molecular weight gradient expands into the homogenous region below, with decreasing velocity (proportional to the decreasing gradient), leaving behind a (nearly) homogenous layer. This can be described as a global mixing process for the average concentration \bar{c} of heavy elements given by a solution of the type $\bar{c} \propto \exp(-t/\tau_{th})$. This behavior is partly analogous to a diffusion process, where an inhomogenous mixture asymptotically approaches a homogenous state in the available volume with a timescale τ_{th} . KRT80 consequently define a diffusion coefficient and the related timescale for a typical length L of the instable region as

$$D_{th} = C_{th} \frac{4acT^3}{3c_p\kappa\rho^2} \left[\frac{-\nabla\mu}{\nabla_{ad} - \nabla} \right] = C_{th} \alpha \frac{1}{R_0} \quad \text{and} \quad \tau_{th} \approx \frac{L^2}{D_{th}}. \quad (1)$$

Here C_{th} is a calibration constant, ∇ are logarithmic gradients, and the other symbols have their usual thermodynamic meanings. The whole discussion in KRT80 implies that the thermohaline instability is a one-time event, which leads from an unstable stratification to a homogenous mixture with a calculable timescale. *KRT80 never define nor use a local diffusion velocity.*

3. Application of the thermohaline instability in Deal et al. (2013, =DDVV13)

We see several problems in the way the instability is applied and treated in this paper.

3.1. Local vs. global mixing, one-time event vs. continuous?

Thermohaline instability is a one-time instability leading to a global mixing with no well-defined local diffusion velocity. DDVV13 (and earlier papers) extend this to a *continuous and local* process by defining a local thermohaline diffusion velocity as

$$v_{th} = D_{th} \frac{\partial \ln c}{\partial r} = -\frac{D_{th}}{H_p} \frac{\partial \ln c}{\partial \ln p} \quad (2)$$

and combining this with continuous accretion and molecular diffusion. Adding v_{th} acting on the bulk material and the molecular diffusion velocity v_{12} , which is a relative velocity between a heavy atom and the main bulk material, different for each species, can be regarded as a technical trick to facilitate computations. It is meaningful only in the two limiting cases, where either of the two processes is negligible. The analogy with convection, where this method of combining “convective mixing” and diffusion is also sometimes used, is misleading: the mixing coefficient in the dynamically unstable convection zone is always orders of magnitude larger than any molecular diffusion, and it is zero outside the cvz (+ overshooting region). A combination of the two mixing processes therefore has never any practical importance.

3.2. Definition of the thermohaline diffusion coefficient

Widely differing coefficient have been used in the literature. DDVV13 state that their coefficient is calibrated with numerical simulations by Traxler et al. (2011). That paper defines a purely empirical fit without physical basis, determined at Prandtl and Lewis numbers of ≈ 0.1 , whereas in the astrophysical context (see below) these numbers are $\approx 10^{-7} - 10^{-8}$, requiring an extrapolation of a numerical fit over seven orders of magnitude.

The thermohaline instability applies only in a region where $\nabla < \nabla_{ad} + \nabla_{\mu}$ (with $\nabla_{\mu} < 0$). This is the limit of dynamical instability in case of a negative μ gradient (the Ledoux criterion for convection). The expression in brackets in eq. 1 ($= 1/R_0$) goes to the limit 1, when approaching this critical gradient. *In stark contrast to this behavior, DDVV13 replace $1/R_0$ by an expression, which has an infinite singularity at the limit of the dynamically stable region.* This virtually assures an instability at the boundary for arbitrary small μ gradients and cannot be physically sound. We therefore use the KRT80 formulation in our numerical estimates.

4. Numerical example

Although we have doubts in the validity of the DDVV13 approach, we calculate the conditions at the bottom of the convection zone (cvz) in the well-studied DA white dwarf G29-38 (11820 K, $\log g = 8.4$), following their equations with the exception of the singularity in D_{th} . Here we use the original KRT80 formulation.

The thermohaline diffusion velocity is $v_{th} \propto c$. For small number concentrations of heavy elements c , therefore $v_{th} \ll v_{12}$, the molecular diffusion velocity, which is independent of c . Likewise, the growth time of the linear instability is much larger

than the molecular diffusion timescale. In the beginning of an accretion event therefore the diffusion equilibrium will be reached before the instability develops. We have thus calculated the abundance distribution for this equilibrium and determined the condition for thermohaline mixing for this stratification.

Primary condition for instability is $R_0 < 1/Le$, which expresses the condition that the excess heat is lost faster than the particles diffuse. Only for the largest abundances is this condition fulfilled in a small region below the cvz in G29-38, which has one of the highest abundances observed in DAs. For total metal abundances below 10^{-7} the instability would never occur. Very similar conclusions can be reached comparing the diffusion velocities (Fig. 1).

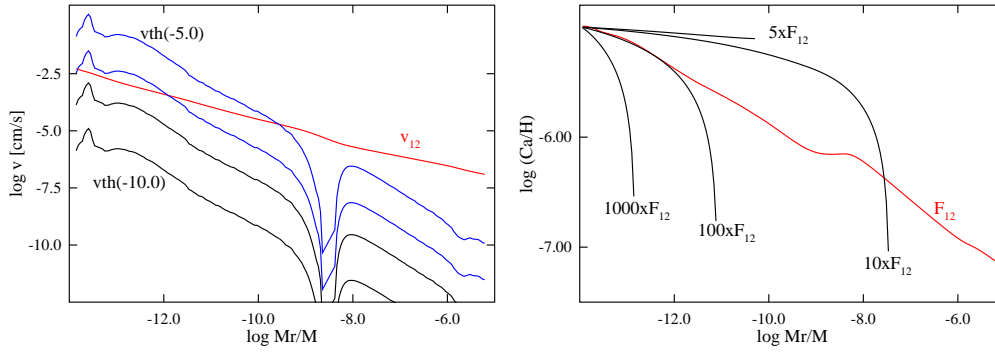


Figure 1. Left: molecular (v_{12}) and thermohaline (v_{th}) diffusion velocities in the outer layers for various metal abundances $\log \text{Ca}/\text{H}$ from -5 to -10. Right: Searching for steady state solutions for the heavy element abundance using only molecular diffusion (F_{12}) or including thermohaline diffusion with a flux increased by large factors.

5. No steady state solution with thermohaline mixing

A steady state solution is characterized by a constant flux F of trace heavy particles at all layers. The sum of thermohaline mixing, ordinary diffusion, and gravitational settling is

$$a_1 \left(\frac{dc}{d \ln p} \right)^2 + a_2 \left(\frac{dc}{d \ln p} \right) + \rho v_{12} c = F = \text{const} \quad (3)$$

with coefficients a_1, a_2 independent of c . We solve this equation with the starting value c at the bottom of the cvz and F as a free parameter. For $F = \rho v_{12} c$ the standard steady state solution with molecular diffusion alone is recovered. For larger accretion fluxes the concentration gradient gets steeper with increasing F , inevitably leading to $c = 0$ or complex values (Fig. 1). As a result *there is no steady state solution including thermohaline mixing*. We assume that the abundance in the cvz will continue to rise until it reaches the “standard” value for molecular diffusion alone, but this needs confirmation by time-dependent calculations up to the final steady state.

6. Astrophysical support

Attempts to explain astrophysical phenomena (e.g. the abundance patterns on the giant branches) have met with only limited success (Cantiello & Langer 2010; Wachlin et al. 2011; Théado & Vauclair 2012), and often an increase of the mixing efficiency by large factors, or invoking additional mixing processes, is required to get agreement with observations.

The differences in maximum accretion rates between DA and DB white dwarfs used by DDVV13 as confirmation of their calculations have a much more plausible explanation in the uncertainties of the EOS for liquid helium. On the other hand, a result from Koester et al. (2014) lends support to our arguments. We find that the range of derived accretion rates in DA white dwarfs remains the same between $10^{5.5}$ to $10^{8.5}$ g/s over the entire observed range of T_{eff} from 5000 to 25000 K, cooling times from 20 Myr to 2 Gyr, diffusion timescales from days to 100000 yrs, from purely radiative envelopes to very deep convection zones, and observed Ca or Si abundances from 10^{-12} to $10^{-4.5}$. It is highly unlikely that such a consistent result would be achieved with the inclusion of a thermohaline mixing description as in DDVV13.

7. Conclusions

Thermohaline instability may play an important rôle in astrophysics. Possible examples are the cases discussed in KRT80 with differences of the molecular weight of the order of 1, or catastrophic events like the sudden infall of a $0.03 M_{jup}$ object on a star (Theado & Vauclair 2012). Concerning the accretion on white dwarfs, with a gradual build-up of heavy element abundances and μ differences of 10^{-6} or smaller, the situation is quite different.

The validity of extending the instability to a continuous process, the mixing with molecular diffusion, and the extrapolation of mixing efficiencies over seven orders of magnitude in Lewis numbers, is not obvious. The constancy of derived accretion fluxes in DAs over an extreme range of all parameters is a strong argument against the importance of the thermohaline instability in this scenario.

A more detailed version of this study with more figures can be found at www1.astrophysik.uni-kiel.de/~koester/astrophysics/astrophysics.html

References

- Cantiello, M., & Langer, N. 2010, A&A, 521, A9. **1006.1354**
- Deal, M., Deheuvels, S., Vauclair, G., Vauclair, S., & Wachlin, F. C. 2013, A&A, 557, L12. **1308.5406**
- Kippenhahn, R., Ruschenplatt, G., & Thomas, H.-C. 1980, A&A, 91, 175
- Koester, D., Gänsicke, B. T., & Farihi, J. 2014, A&A, 566, A34. **1404.2617**
- Théado, S., & Vauclair, S. 2012, ApJ, 744, 123. **1109.4238**
- Traxler, A., Garaud, P., & Stellmach, S. 2011, ApJ, 728, L29. **1011.3461**
- Ulrich, R. K. 1972, ApJ, 172, 165
- Wachlin, F. C., Miller Bertolami, M. M., & Althaus, L. G. 2011, A&A, 533, A139. **1104.0832**